A Semantic Dissimilarity Measure for Concept Descriptions in Ontological Knowledge Bases

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Abstract. This work presents a dissimilarity measure for expressive Description Logics that are the theoretical counterpart of the standard representations for ontological knowledge. The focus is on the definition of a dissimilarity measure for the $\mathcal{ALC}$ description logic based both on the syntax and on the semantics of the descriptions.

1 Introduction

Recent investigations have emphasized the use of ontologies similarity measures for *Information Retrieval* and *Integration* [1, 2]. However, there is a number of other tasks that may exploit similarity measures, such as, for instance, clustering by means of *partitional* or *agglomerative* algorithms. Therefore, in a Semantic Web perspective, similarity measures can enable such algorithms to exploit the available ontological knowledge expressed in suitable representations, namely concept languages which are candidate as standard in this context.

Various measures for concept representations have been proposed in the literature. A measure has been proposed as a function of the *path distance* between terms in the hierarchical structure underlying the ontology [3]. Other methods for assessing the similarity of concept descriptions are based on *feature matching* [4] and *information content* [5]. The former approach uses both common and discriminant features among concepts and/or concept instances to compute the semantic similarity. The latter method is founded on *Information Theory*. A similarity measure for concepts within a hierarchy is defined in terms of the amount of information conveyed by their immediate super-concept. This is a measure of the variation of information from a description level to a more general one.

Other measures compute the similarity among classes (concepts) belonging to different ontologies. In [6] a number of measures is presented for comparing concepts located in possibly heterogeneous ontologies. The following requirements are made: the formal representation supports inferences such as *subsumption* and local concepts in different ontologies inherit their definitional structure from concepts in a shared ontology. In particular, the intersection of the sets of concept instances is assumed as an indication of the correspondence between these concepts. In [7] a similarity function determines similar classes by using a matching process making use of synonym sets, semantic neighborhood, and discriminating features that are classified into parts, functions, and attributes.

Most of the cited works adopt a semantic approach in conjunction with the structure of the considered descriptions. Besides, the syntactic structure of the descriptions becomes much less important when richer representations are adopted due to the expressive operators that can be employed.

Most of these works focussed on the similarity of atomic concepts (within a hierarchy) rather than on composite ones. Nevertheless, the standard ontology markup languages (e.g., OWL) are founded in Description Logics (DLs) since they borrow the typical DLs constructors. Thus, it becomes necessary to investigate the similarity of complex concept descriptions expressed in DLs. In this respect, to the best of our knowledge, there has been no comparable effort in the literature, except the ideas in [8].

In this position paper, we introduce a semantic dissimilarity measure between descriptions which is suitable for an expressive DL like $\mathcal{ALC}$ [9]. The measure is based on the underlying semantics elicited by querying the knowledge base, as proposed also in [10]. Moreover, recurring the notion of *most specific concept* of an individual, the measure is extended to the individual-concept and individual-individual cases, which may be exploited in knowledge discovery settings.

2 The Reference Representation Language

The basics of $\mathcal{ALC}$ [9] are recalled, a logic which is sufficiently expressive to support most of the constructors of standard ontology languages.

Primitive *concepts*, denoted with names from $N_C = \{C, D, \ldots\}$, are interpreted as subsets of a domain of objects and primitive *roles*, denoted with names taken from $N_R = \{R, S, \ldots\}$, are interpreted as binary relations on such a domain. Complex descriptions are built using primitive concepts and roles and the constructors in Table 1. The meaning is defined by an *interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where $\Delta^{\mathcal{I}}$ is the *domain* of the interpretation and $\cdot^{\mathcal{I}}$ is the *interpretation function*, mapping the intension of concepts and roles to their extension.

A *knowledge base* $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ contains a *T-box* $\mathcal{T}$ and an *A-box* $\mathcal{A}$. $\mathcal{T}$ is a set of definitions $C \equiv D$, meaning $C^{\mathcal{I}} = D^{\mathcal{I}}$, where C is the concept name and D is a description as defined above. $\mathcal{A}$ contains extensional assertions on concepts and roles, e.g. $C(a)$ and $R(a, b)$, meaning, resp., that $a^{\mathcal{I}} \in C^{\mathcal{I}}$ and $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$.

Definition 2.1. *Given two concept descriptions C and D, C* subsumes *D, denoted by $C \sqsupseteq D$, iff for every interpretation $\mathcal{I}$ it holds that $C^{\mathcal{I}} \supseteq D^{\mathcal{I}}$.*

Example 2.1. An instance of concept definition in the proposed language is: $\mathsf{Father} \equiv \mathsf{Male} \sqcap \exists \mathsf{hasChild}.\mathsf{Person}$ which corresponds to the sentence: *"a father is a male (person) that has some persons as his children"*. The following are instances of simple assertions: $\mathsf{Male}(\mathsf{Leonardo})$, $\mathsf{Male}(\mathsf{Vito})$, $\mathsf{hasChild}(\mathsf{Leonardo}, \mathsf{Vito})$.

Supposing that $\mathsf{Male} \sqsubseteq \mathsf{Person}$ is known (in the T-Box), one can deduce that: $\mathsf{Person}(\mathsf{Leonardo})$, $\mathsf{Person}(\mathsf{Vito})$ and then $\mathsf{Father}(\mathsf{Leonardo})$.

Given these primitive concepts and roles, it is possible to define many other related concepts: $\mathsf{Parent} \equiv \mathsf{Person} \sqcap \exists \mathsf{hasChild}.\mathsf{Person}$ and $\mathsf{FatherWithoutSons} \equiv$

Table 1. $\mathcal{ALC}$ constructors and their meaning.

Name	Syntax	Semantics
top concept	$\top$	$\Delta^{\mathcal{I}}$
bottom concept	$\bot$	$\emptyset$
concept	C	$C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
concept negation	$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
concept conjunction	$C_1 \sqcap C_2$	$C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}}$
concept disjunction	$C_1 \sqcup C_2$	$C_1^{\mathcal{I}} \cup C_2^{\mathcal{I}}$
existential restriction	$\exists R.C$	$\{x \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}}((x,y) \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}})\}$
universal restriction	$\forall R.C$	$\{x \in \Delta^{\mathcal{I}} \mid \forall y \in \Delta^{\mathcal{I}}((x,y) \in R^{\mathcal{I}} \rightarrow y \in C^{\mathcal{I}})\}$

$\mathsf{Male} \sqcap \exists \mathsf{hasChild.Person} \sqcap \forall \mathsf{hasChild.}(\neg \mathsf{Male})$. It is easy to see that the following relationships hold: $\mathsf{Parent} \sqsupseteq \mathsf{Father}$ and $\mathsf{Father} \sqsupseteq \mathsf{FatherWithoutSons}$. □

A related inference used in the following is *instance checking*, that is deciding whether an individual is an instance of a concept [9]. Conversely, it may be necessary to solve the *realization problem* that requires finding the concepts which an individual belongs to, especially the most specific one, if any:

Definition 2.2. *Given an A-Box $\mathcal{A}$ and an individual a, the* most specific concept *of a w.r.t. $\mathcal{A}$ is the concept C, denoted $\mathsf{MSC}_{\mathcal{A}}(a)$, such that $\mathcal{A} \models C(a)$ and $C \sqsubseteq D$, $\forall D$ such that $\mathcal{A} \models D(a)$.*

In the general case of a cyclic A-Box expressed in a DL endowed with existential or numeric restriction the MSC cannot be expressed as a finite description [9], thus it can only be approximated. Generally an approximation of the MSC is considered up to a certain depth k. The maximum depth k has been shown to correspond to the depth of the considered A-Box [11].

Especially for rich DL languages such as $\mathcal{ALC}$, many semantically equivalent (yet syntactically different) descriptions can be given for the same concept. Nevertheless, equivalent concepts can be reduced to a normal form by means of rewriting rules that preserve their equivalence [9]:

Definition 2.3. *A concept description D is in $\mathcal{ALC}$* normal form *iff $D \equiv \bot$ or $D \equiv \top$ or if $D = D_1 \sqcup \cdots \sqcup D_n$ $(\forall i = 1, \ldots, n,\ D_i \not\equiv \bot)$ with*

$$D_i = \bigsqcap_{A \in \mathsf{prim}(D_i)} A \ \sqcap \bigsqcap_{R \in N_R} \left[\forall R.\mathsf{val}_R(D_i) \sqcap \bigsqcap_{E \in \mathsf{ex}_R(D_i)} \exists R.E \right]$$

where: $\mathsf{prim}(C)$ is the set of all (negated) primitives occurring at the top level of C; $\mathsf{val}_R(C)$ is the conjunction $C_1 \sqcap \cdots \sqcap C_n$ in the value restriction of role R, if any (otherwise $\mathsf{val}_R(C) = \top$); $\mathsf{ex}_R(C)$ is the set of concepts in the value restriction of the role R.
For any R, every sub-description in $\mathsf{ex}_R(D_i)$ and $\mathsf{val}_R(D_i)$ is in normal form.

3 A Dissimilarity Measure for $\mathcal{ALC}$

As a first step we need to define a dissimilarity measure for $\mathcal{ALC}$ descriptions. In order to achieve this goal, we introduce a function which is necessary for the correct definition of a dissimilarity measure. This should be a definite positive function on the set of $\mathcal{ALC}$ normal form concept description, defined making use of the syntax and semantics of the concepts (and roles) involved in the descriptions. The function is formally defined as follows:

Definition 3.1. *Let $\mathcal{L} = \mathcal{ALC}/_{\equiv}$ be the set of all concepts in $\mathcal{ALC}$ normal form and let $\mathcal{A}$ be an A-Box with canonical interpretation $\mathcal{I}$. f is a function $f : \mathcal{L} \times \mathcal{L} \mapsto R^+$ defined as follows:*
for all $C, D \in \mathcal{L}$, with $C = \bigsqcup_{i=1}^{n} C_i$ and $D = \bigsqcup_{j=1}^{m} D_j$

$$f(C, D) := f_{\sqcup}(C, D) = \begin{cases} 1 & \text{if } C \equiv D \\ 0 & \text{if } C \sqcap D \equiv \bot \\ \max_{\substack{i=1,\dots,n \\ j=1,\dots,m}} f_{\sqcap}(C_i, D_j) & \text{otherwise} \end{cases}$$

$$f_{\sqcap}(C_i, D_j) := f_P(\mathsf{prim}(C_i), \mathsf{prim}(D_j)) + f_{\forall}(C_i, D_j) + f_{\exists}(C_i, D_j)$$

$$f_P(\mathsf{prim}(C_i), \mathsf{prim}(D_j)) := \frac{|(\mathsf{prim}(C_i))^{\mathcal{I}} \cup (\mathsf{prim}(D_j))^{\mathcal{I}}|}{|(\mathsf{prim}(C_i))^{\mathcal{I}} \cap (\mathsf{prim}(D_j))^{\mathcal{I}}|}$$

yet, $f_P(\mathsf{prim}(C_i), \mathsf{prim}(D_j)) = 0$ when $(\mathsf{prim}(C_i))^{\mathcal{I}} \cap (\mathsf{prim}(D_j))^{\mathcal{I}} = \emptyset$

$$f_{\forall}(C_i, D_j) := \sum_{R \in N_R} f_{\sqcup}(\mathsf{val}_R(C_i), \mathsf{val}_R(D_j))$$

$$f_{\exists}(C_i, D_j) := \sum_{R \in N_R} \sum_{k=1}^{N} \max_{p=1,\dots,M} f_{\sqcup}(C_i^k, D_j^p)$$

where $C_i^k \in \mathsf{ex}_R(C_i)$ and $D_j^p \in \mathsf{ex}_R(D_j)$ and we suppose w.l.o.g. that $N = |\mathsf{ex}_R(C_i)| \geq |\mathsf{ex}_R(D_j)| = M$, otherwise the indices N and M are to be exchanged in the formula above.

The function f represents a measure of the overlap between two descriptions (namely C and D) expressed in $\mathcal{ALC}$ normal form. It is defined recursively beginning from the top level of the descriptions (a disjunctive level) up to the bottom level represented by (conjunctions of) primitive concepts.

In case of disjunction, the overlap between the two concepts is equal to the maximum of the overlaps calculated among all couples of disjuncts (C_i, D_j) that make up the top level of the considered concepts.

Then, since every disjunct is a conjunction of descriptions, it is necessary to calculate the overlap between conjunctive concepts. This is calculated as the sum of the overlap among the parts that make up the conjunctive description. Specifically, a conjunctive form can have three different types of terms: primitive concepts, universal restrictions and existential restrictions. Since conjunction ($\sqcap$) is a symmetric operator, it is possible to put together every type of restriction (occurring at the same level) so it is possible to consider the conjunctions of primitive concepts, the conjunctions of existential restrictions and the conjunction of universal restrictions as specified in the definition of $\mathcal{ALC}$ normal form.

Next, the amount of the overlap for the three different type of conjunction is defined. Particularly, the amount of overlap between two conjunctions of (negated) primitive concepts is null if the they do not share any individual in their extension. Conversely, if the two concepts share some individual the overlap between them is computed as the ratio between the union and the intersection of their extensions which expresses how far the partial overlap is from the total overlap of the two concepts.

The computation of the overlap between, resp., descriptions expressed by universal and existential restrictions is a bit more complex. Considering the conjunction of universal restrictions, it is worthwhile to recall that every such restriction is a single conjunction linked by respect to a different role (since $\forall R.C \sqcap \forall R.D \equiv \forall R.(C \sqcap D)$). Moreover, the scope of each restriction is expressed in normal form. Thus, the amount of the overlap between two subconcepts (within C_i and D_j, resp.) that are scope of a universal restriction on a certain role R is given by the overlap between two concepts in normal form (computed by $f_\sqcup$); of course, if no disjunction occurs at the top level, it is possible to regard the concept description as a disjunction of a single term to which $f_\sqcup$ applies in a simple way. Since one may have a conjunction of concepts with universal restrictions, one per different role, the overlap of this conjunction is given by the sum of the overlap yielded by each restriction, rather than every restriction scope. Note that, when a universal restriction on a role occurs only in one of the descriptions, then the computation assumes $\top$ as the corresponding concept in the other description.

Now we turn to analyze the computation of the amount of the overlap between two descriptions made up of conjunctions of existential restrictions. For the dissimilarity between existential restrictions, we may recur to *existential mappings*. Supposing that $N = |\mathsf{ex}_R(C_i)| \geq M = |\mathsf{ex}_R(D_j)|$, such a mapping can be defined as a function $\alpha : \{1, \ldots, N\} \mapsto \{1, \ldots, M\}$. If each element of $\mathsf{ex}_R(C_i)$ and $\mathsf{ex}_R(D_j)$ is indexed with an integer in the ranges $[1, N]$ and $[1, M]$, resp., then any function α maps each concept description $C_i^k \in \mathsf{ex}_R(C_i)$ to $D_j^p \in \mathsf{ex}_R(D_j)$. Since each C_i^k (resp. D_j^p) is in normal form, it is possible to calculate the amount of their overlap using $f_\sqcup$. Fixed a role R and considered a certain C_i^k (with $k \in [1, N]$), the amount of the overlap between C_i^k and D_j^p (with $p \in [1, M]$) is computed. We are supposing that $N \geq M$, thus each existential restriction on role R is coupled with the one on the same role in other description scoring the maximum amount of overlap. These maxima are summed up per single role. In case of absence of role restrictions on a certain role from either description then it is considered as the concept $\top$.

Summing up, we have defined a measure whose baseline (counts on the extensions of primitive concepts) depends on the semantics of the knowledge base, as conveyed by the ABox assertions. This is in line with to the ideas in [10, 8], where semantics is elicited as a probability distribution over the domain of the interpretation Δ.

Now, it is possible to derive a dissimilarity measure based on f as follows

Definition 3.2. *Let $\mathcal{L}$ be the set of descriptions in $\mathcal{ALC}$ normal form and let f be an overlap function defined as above. The* dissimilarity measure *d is a function $d : \mathcal{L} \times \mathcal{L} \mapsto [0, 1]$ such that, for all $C = \bigsqcup_{i=1}^{n} C_i$ and $D = \bigsqcup_{j=1}^{m} D_j$ concept descriptions in $\mathcal{ALC}$ normal form:*

$$d(C, D) := \begin{cases} 1 & \text{if } f(C, D) = 0 \\ 0 & \text{if } f(C, D) = 1 \\ \frac{1}{f(C,D)} & \text{otherwise} \end{cases}$$

The function d measures the level of dissimilarity between two concepts, say C and D, in $\mathcal{ALC}$ normal form using the function f that expresses the amount of overlap between the two concepts. Particularly, if $f(C, D) = 0$ then this means that there is no overlap between the considered concepts, therefore d must indicate that the two concepts are totally different, indeed $d(C, D) = 1$ i.e. it amounts to the maximum value of its range. If $f(C, D) = 1$ this means that the two concepts are totally overlapped and consequently $d(C, D) = 0$ that means that the two concept are undistinguishable, indeed d assumes the minimum value of its range. If the considered concepts have a partial overlap then their dissimilarity is lower as much as the two concept are more overlapped, since in this case $f(C, D) > 0$ and consequently $0 < d(C, D) < 1$.

Let us recall that, for every individual in the A-Box, it is possible to calculate the most specific concept of an individual a w.r.t. an A-box, $\mathsf{MSC}(a)$ (see Def. 2.2) or at least its approximation $\mathsf{MSC}^k(a)$ up to a certain description depth k. In some cases these are equivalent concepts but in general $\mathsf{MSC}^k(a) \sqsupseteq \mathsf{MSC}(a)$. This notion is exploited to lift the individuals to the concept level.

Let a and b two individuals in a given A-Box. We can consider $A^* = \mathsf{MSC}^k(a)$ and $B^* = \mathsf{MSC}^k(b)$ (we also suppose that they are in $\mathcal{ALC}$ normal form). Now, in order to assess the dissimilarity between the considered individuals, the dissimilarity measure d can be applied to these descriptions, as follows:

$$d(a, b) := d(A^*, B^*) = d(\mathsf{MSC}^k(a), \mathsf{MSC}^k(b))$$

Analogously, the dissimilarity value between a concept description C and an individual a can be computed by determining the MSC approximation of the individual and then applying the dissimilarity measure:

$$\forall a : \ d(a, C) := d(\mathsf{MSC}^k(a), C)$$

This case may turn out to be particularly handy both in inductive reasoning (construction, repairing of knowledge bases) and in information retrieval.

We prove that d function actually is a dissimilarity measure (or *dissimilarity function* [12]), according to the following formal definition:

Definition 3.3. *Let* S *be a non empty set of elements. A* dissimilarity measure *for S is a real-valued function r defined on the set $S \times S$ that fulfills the properties:*
1. $r(a,b) \geq 0,\ \forall a,b \in S$ *(positive definiteness);*
2. $r(a,b) = r(b,a),\ \forall a,b \in S$ *(symmetry);*
3. $\forall a,b \in S\colon\ r(a,b) \geq r(a,a)$

Proposition 3.1. *The function d is a dissimilarity measure on $\mathcal{L} = \mathcal{ALC}/_{\equiv}$.*

Proof.
1. *trivial: by construction d computes dissimilarity by using sums of positive quantities and maxima computed on sets of such values.*
2. *by the commutativity of the sum and maximum operators.*
3. *by the definition of d, it holds that $d(C,C) = 0$ and $d(C,C') = 0$ if C is semantically equivalent to C'. In all other different cases, $\forall D \in \mathcal{L}$ and D not semantically equivalent to D ($C \not\equiv D$), we have: $d(C,D) > 0$* □

The computational complexity of our dissimilarity measure d is strictly related to that of f. The measure also relies on some reasoning services, namely subsumption and instance-checking, therefore its complexity depends on the complexity of these inferences too. In order to assess the complexity of d, we distinguish three different cases descending from being d based on $f_{\sqcup}$.
Let $C = \bigsqcup_{i=1}^{n} C_i$ and $D = \bigsqcup_{j=1}^{m} D_j$ be two descriptions in normal form:
– *Case 1: C and D are semantically equivalent.* Only subsumption is involved in order to verify the semantic equivalence of the concepts. Thus $\mathit{Compl}(d) = 2 \cdot \mathit{Cmpl}(\sqsupseteq)$, where $\mathit{Cmpl}(\cdot)$ and $\sqsupseteq$ represent, resp., complexity and subsumption;
– *Case 2: C and D are disjoint yet not semantically equivalent.* Subsumption and conjunction are involved. Being the time complexity of conjunction a constant, we have the same complexity of the previous case
– *Case 3: C and D are not semantically equivalent nor disjoint.* The complexity depends on the structure of the concepts. It is necessary to compute $f_{\sqcap}$ for $n \cdot m$ times; so the complexity is: $\mathit{Cmpl}(d) = nm \cdot \mathit{Cmpl}(f_{\sqcap}) = nm \cdot [\mathit{Cmpl}(f_P) + \mathit{Cmpl}(f_{\forall}) + \mathit{Cmpl}(f_{\exists})]$. Thus we analyze the complexity of f_P, $f_{\forall}$, $f_{\exists}$.
The dominant operation when computing f_P is instance checking (IC) used for determining the concept extensions. So we conclude that $C(f_P) = 2 \cdot C(IC)$. The computation of $f_{\forall}$ and $f_{\exists}$ apply recursively the definition of $f_{\sqcup}$ on less complex descriptions. A maximum of $|N_R|$ calls of $f_{\sqcup}$ are needed for computing $f_{\forall}$, while the calls of $f_{\sqcup}$ needed for $f_{\exists}$ are $|N_R| \cdot N \cdot M$, where $N = |\mathsf{ex}_R(C_i)|$ and $M = |\mathsf{ex}_R(D_j)|$ as in Def. 3.1. Summing up as in the previous equation:
$\mathit{Cmpl}(d) = nm \cdot [(2 \cdot \mathit{Cmpl}(IC)) + (|N_R| \cdot \mathit{Cmpl}(f_{\sqcup})) + (|N_R| \cdot M \cdot N \cdot \mathit{Cmpl}(f_{\sqcup}))]$

We conclude that the complexity of the computation of d depends on the complexity of the instance-checking for $\mathcal{ALC}$ which is P-space [9]. Nevertheless, in practical applications, these computations may be efficiently carried out exploiting the statistics that are maintained by the DBMSs query optimizers. Besides, the counts that are necessary for computing the concept extensions could be estimated by means of the probability distribution over the domain.

4 Conclusions and Further Developments

Similarity measures turn out to be useful in several tasks such as, classification, case-based reasoning, clustering, etc. A novel dissimilarity measure d has been introduced, derived from the measure f of the overlap between $\mathcal{ALC}$ descriptions, and based on the underlying semantics based on ABox interpretation.

We have also shown how to apply this function to measuring the dissimilarity between individuals and also a individual-concept dissimilarity, which may be more useful in knowledge discovery tasks.

In particular, defining a measure that is applicable for both the concepts to individual similarity and between individuals one, it is suitable for agglomerative clustering and for divisional clustering too. A further investigation will concern the derivation of a distance measure, which amounts to finding a measure that fulfils the triangular property.

These ideas are being exploited also for defining kernels on rich representations like DLs, thus allowing the exploitation of the efficiency of SVMs and the other related methods.

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